OPTIMIZATION OF MILK PROCESSING PROCESSES AND ANALYSIS OF OBTAINED SOLUTIONS

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ABSTRACT
This study focuses on the analysis of optimization and sensitivity in dairy production, utilizing linear programming to achieve an optimal production structure and maximize net income. The model identifies key products with the highest revenues and those having the greatest impact on overall revenue and costs. Sensitivity analysis reveals which products are most sensitive to price changes and how these changes affect production structures, providing valuable insights for dairy management in pricing strategies and future investments. Additionally, the research opens opportunities for further improvements and exploring new market avenues and product diversification in the dairy industry.

Keywords: milk, optimization, linear programming, net income, production structure, sensitivity analysis

JEL: C61, Q13, D24, Q10

Introduction
Daily changes in market conditions emphasize the need for thorough planning and organization in agriculture and agribusiness (Nedeljković et al., 2023). Introducing new decisions in the processing sector has become crucial in response to the dynamic market environment and for profit maximization. Stimulating producers and creating added value for consumers represent opportunities for stable revenues (Ivanović and Ćosić, 2023), crucial for maximizing profits in the processing sector, with the possibility of high-quality raw milk for competitive products.

Grujić and colleagues (2023) highlight that in the period from 2012 to 2021, the majority of investments in production amounted to 25.3% in the Republic of Serbia,
which encourages support for the production sector, improvement of technology and capacity, and increasing the availability of raw materials for milk processing. Stable and sustainable production requires acquiring new knowledge and applying positive practices (Čekrlija et al., 2023), crucial for successfully facing the challenges of the modern market. Sustainable production contributes to long-term stability and profit maximization in the processing industry, ensuring long-term supply of raw milk.

The continuous decline in the quality of food products throughout the food supply chain, as noted in the studies by Bloemhof and Soysal (2017), along with the challenges posed by the COVID-19 pandemic, such as disruptions in milk supply chains (Abbasian et al., 2023), underscores the need to consider alternative strategies to reduce losses and preserve product quality. In this context, increasing the processing of raw milk in dairies may be one solution. Milk processing in dairies allows for longer preservation of freshness and extends the shelf life of dairy products. Additionally, processing in dairies can provide better product quality control and reduce the risk of losses occurring in the distribution chain of raw milk. This can, in turn, reduce milk waste and enhance the efficiency of the supply chain.

Agriculture is facing a crucial challenge in making food production more sustainable. This requires the implementation of new production systems, technologies, tools, and programs on a large scale, involving significant changes in existing practices of agricultural production, processing, and food markets (Springmann et al., 2018; Möhring et al., 2023). All of this underscores the importance of continuous development and optimization of supply chains, especially in light of the COVID-19 pandemic, as emphasized in the works of Meyer et al. (2021) and Sharma et al. (2023). With the development of information technology, optimization methods are increasingly relying on software packages, becoming fundamental tools with the aid of computers and computer applications, such as the use of Solver tool.

**Literature review**

Introduction of linear programming into agricultural practices, especially in activities such as transportation, production, and distribution (Kelaba, 2021), has brought about a more efficient resource management strategy. Mathematical optimization, the foundation of linear programming application, is recognized as a key tool for addressing various agricultural problems (Djukić, 2023). This approach enables the minimization or maximization of linear functions while adhering to linear constraints.

Furthermore, research conducted by Maddah and colleagues (2021) indicates that retailers often have multiple priorities beyond profit alone, highlighting the need for a more comprehensive decision-making approach. Consistent with this, Addis and colleagues (2021) emphasize the usefulness of linear programming in identifying optimal solutions for retailers.

The growing attention towards sustainable food supply chains (Qahtan et al., 2023) motivates researchers to explore new approaches, particularly in the context of the 4.0
revolution in agriculture and the food industry. In modern agriculture, where decision-making based on real facts has become imperative (Vico et al., 2017), production optimization is crucial for the sustainable growth of manufacturing enterprises (Dejanović et al., 2019). The application of operational research, including linear programming, enables rational resource utilization (Kelava, 2021), thereby contributing to agricultural growth and increasing GDP contributions (Kamath, 2020). In this context, optimization and simulation models provide the opportunity to understand the optimal structure of production and study system behavior under different conditions (Orović et al., 2015).

Given the assertion that issues in small-scale agricultural production include fragmentation and the lack of connectivity with processing industries (Vico et al., 2022), this paper will further analyse another model addressed by Jandrić (2019), a model for optimizing milk production (processing) in dairy factories, enabling efficient resource and process management to achieve the best results in the quality and quantity of dairy products produced. The model demonstrates potential for further improvements, particularly in integration with quality monitoring and production sustainability systems. The aim is to thoroughly analyse the model, considering various factors affecting processes in the dairy factory, including resources, technologies, and economic feasibility. Through this analysis, the authors seek to identify opportunities for improvement and innovation, offering suggestions for more efficient and sustainable dairy product production. Such a focus allows for a deeper exploration of the complexity of processes in the dairy factory and provides insights into changes that would enhance the milk value chain.

Input prices from 2019 were utilized in this study, reflecting the period when the dissertation was conducted, to maintain consistency in the analysis and highlight the importance of optimization in achieving optimal outcomes. Optimization can help users recognize opportunities to improve resource efficiency, reduce costs, and enhance production quality. Through precise modeling and analysis, this paper emphasizes how linear programming enables flexibility and accuracy in evaluating various scenarios and making decisions in milk production.

According to Jandrić, the development of a model for optimizing milk production (processing) in a dairy is based on technological, organizational, and economic data provided by the surveyed dairy. A systemic approach was applied in creating the model, which includes the development of a logical and mathematical model, taking into account the key characteristics of the observed production system.

**Mathematical model of milk production optimization in dairy**

**Objective function**

$$(\text{max}) f = \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} x_{ij}$$

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Constraints

\[
\sum_{i=1}^{p} \sum_{j=1}^{q} a_{ijkl}x_{ij} \leq \geq u_{kl} \quad k=1,2,\ldots,r \quad l=1,2,\ldots,s
\]

Non-negativity condition: \(i=1,2,\ldots,p \quad j=1,2,\ldots,q\); Indices: \(p\) - number of activity groups; \(q\) - number of activities in a group; \(r\) - number of constraint groups; \(s\) - number of constraints in a group; Activities: \(x_{ij}\); Indices: \(p\) - number of activity groups; \(q\) - number of activities in a group; Constraints: \(u_{kl}\); Indices: \(r\) - number of constraint groups; \(s\) - number of constraints in a group; Coefficients in the objective function: \(c_{ij}\); Indices: \(p\) - number of activity groups; \(q\) - number of activities in a group; Coefficients in the constraints: quantity of the \(j\)-activity in the \(i\)-activity group for the \(i\)-constraint in the \(r\)-constraint group.

**Activity groups:** main products; \(i=1 j=6\); by-products \(i=2 j=1,2,\ldots,5\); inputs representing costs, separately listed in the model \(i=3 j=1,\ldots,14\); other costs \(i=4 j=1\); labor expenditure \(i=5 j=1\); finished products \(i=6 j=1,\ldots,11\).

**Constraint groups:** capacities \(k=1 l=1,2\); market constraints \(k=2 l=4\); raw milk balance \(k=3 l=1\); by-products balances \(k=4 l=1,\ldots,5\); balances of inputs representing costs, separately listed in the model \(k=5 l=1,\ldots,13\); other costs balance \(k=6 l=1\); labor force balance \(k=7 l=1\); finished products balance \(k=8 l=1,\ldots,11\).

**Research Results and Discussion**

In the process of creating a linear programming model, whether it pertains to primary agricultural production or the food industry, a crucial step is the systemic analysis of the modeled object, as well as the development of a logical model. When constructing a logical model, it is important to define the most important elements of the system being modeled and their interrelationships. The logical model can be represented visually, for example, through a context diagram or other illustrative forms (Vico and Rajić, 2018). Novković and colleagues (1997) emphasize that the logical model should encompass all key elements and relationships that objectively exist in the system and are relevant to the planning process. Figure 2 depicts the parts of the system under analysis, including their interrelationships. Raw milk is a key input for the production of main and by-products, which are marketed. Besides raw milk, other inputs also influence production and appear as activities or constraints in the mathematical model. Market constraints have been identified, defining minimum or maximum levels of finished product production.
Activities in the milk processing optimization model: Activities, or independent variable quantities (control variables), must meet the fundamental assumptions for the application of linear programming, which include: proportionality, additivity, divisibility, and certainty (Vujošević, 1997).

The model consists of 38 activities grouped into categories: Main products; Pasteurized milk 3.2% fat (X1), White cheese slice 45% fat (X2), Clotted cream 45-50% fat (X3), Yogurt 2.8% fat (X4), Filo cheese pie 20% fat (X5), Sour cream 12% fat (X6), Pepper in cream 35% fat (X7), Fermented whey 0.5% fat (X8), Yogurt 0.5% fat (X9), Bakery cheese (X10), Urda cheese (X11). Inputs representing costs, separately listed in the model; Raw milk (X12), Pepper in brine (X13), Packaging for various products (X14-X24). Other costs; Labor costs (RSD) (X25), Other costs (RSD) (X26). Labor expenditure: Labor expenditure (min) (X27). Finished products; Sales of Pasteurized milk 3.2% fat (X28), Sales of White cheese slice 45% fat (X29), Sales of Clotted cream 45-50% fat (X30), Sales of Yogurt 2.8% fat (X31), Sales of Filo cheese pie 20% fat (X32), Sales of Sour cream 12% fat (X33), Sales of Pepper in cream 35% fat (X34), Sales of Fermented whey 0.5% fat (X35), Sales of Yogurt 0.5% fat (X36), Sales of Bakery cheese (X37), Sales of Urda cheese (X38). These activities represent key parts of the model and cover all aspects of the milk and dairy product production process.

Constraints in the milk processing optimization model: Rajić (2002) emphasizes the importance of selecting appropriate constraints, highlighting that this process is crucial for the realism of the obtained solution. The goal and purpose of the research, as well as other relevant factors, influence the selection of constraints to be used in the model. In creating a model for optimizing milk processing in a dairy, eight groups of constraints were defined in the model for milk processing optimization in the dairy: capacities, market constraints, raw milk balance, by-products balances, balances of other inputs representing costs, balances of other costs, labor expenditure balance, and finished product balance. The first two constraint groups in the model use mathematical
symbols ≤ or ≥, while the other constraint groups are expressed using the “=” symbol with a value of 0. This is due to the specificity of the task of the last five constraint groups in the model, whose purpose is to link activities in the objective function through technical coefficients. When the solution is obtained, their balance, or the value of the constraint, should be zero.

The milk processing optimization model in the dairy includes numerous constraints expressed within different categories: Capacities; Maximum daily capacity of raw milk (Q1), Maximum daily capacity of cheese slices (Q2). Market Constraints; Market constraint for sour cream (1,000 units daily) (Q3), Market constraint for yogurt 2.8% fat (maximum 1,200 liters daily) (Q4), Market constraint for yogurt 0.5% fat (maximum 800 liters daily) (Q5), Market constraint for pasteurized milk 3.2% fat (minimum 400 liters daily) (Q6). Balances; Balance of raw milk (Q7), Balance of cream 35% fat (Q8), Balance of pepper in brine (Q9), Balance of fermented whey (Q10), Balance of yogurt 0.5% fat (Q11), Balance of bakery cheese (Q12), Balance of urda cheese (Q13), Balance of packaging for various products (Q14-Q24), Balance of other costs (Q25), Balance of labor costs (Q26), Balance of labor expenditure (Q27). Sales; Sales of pasteurized milk 3.2% fat (Q28), Sales of white cheese slices 45% fat (Q29), Sales of clotted cream 45-50% fat (Q30), Sales of yogurt 2.8% fat (Q31), Sales of filo cheese pie 20% fat (Q32), Sales of sour cream 12% fat (Q33), Sales of pepper in cream 35% fat (Q34), Sales of fermented whey 0.5% fat (Q35), Sales of yogurt 0.5% fat (Q36), Sales of bakery cheese (Q37), Sales of urda cheese (Q38).

These constraints play a crucial role in ensuring efficient and balanced production of dairy products, with each constraint group contributing to the creation of a realistic and practical optimization model. The maximum daily capacity of raw milk refers to the total amount of raw milk that the dairy can receive in one day, and this capacity is 5,000 liters (Q1). The constraint in the model is expressed as X12≤5,000. The maximum daily capacity of production of white cheese slices is determined by the capacity of the aging room and amounts to 128 kg per day (Q2). The constraint in the model is expressed as X5≤128. The group of market constraints pertains to the minimum and maximum daily quantities of finished products that the dairy can produce. Three products have specified maximum quantities: sour cream (200 units daily), yogurt 2.8% fat (1,200 liters daily), and yogurt 0.5% fat (800 liters daily). Pasteurized milk has a specified minimum daily quantity of 400 liters (Q3-Q6).

Raw milk is the main raw material and is associated with the main products through a specific constraint. The technical coefficients in the equation represent the required quantity of raw milk for one unit of product, which is expressed in the model as: -1.05042X1 - 7.81250X2 - 20.0000X3 - 1.05042X4 - 10.0000X5 - 3.03030X6 + X12 = 0 (Q7).

Additional products are generated in the process of producing main products. Except for sour cream, all main products create one or more additional products. In the process of producing clotted cream, yogurt 0.5% fat and bakery cheese are produced. Pepper in cream is produced during the production of pasteurized milk and yogurt 2.8% fat.
The relationship between main and additional products is shown in the constraint group “Balance of additional/by-products”. Q8: 0.024002X1 + 0.036019X4 – X7 = 0; Q9: -0.428571X7 + X3 = 0; Q10: 3.90625X2 – X8 = 0; Q11: 8.00X3 – X9 = 0; Q12: 2.00X3 – X10 = 0; Q13: -0.40X5 + X11 = 0.

Balances of other cost inputs include 12 constraints (Q14-Q25): Q14: -X1 + X13 = 0; Q15: -X2 + X14 = 0; Q16: -X3 + X15 = 0; Q17: -X4 + X16 = 0; Q18: -X5 + X17 = 0; Q19: -5X6 + X18 = 0; Q20: -1.42857X7 + X19 = 0; Q21: -X8 + X20 = 0; Q22: -X9 + X21 = 0; Q23: -0.20X10 + X22 = 0; Q24: -X11 + X23 = 0; Q25: -0.24X25 + X27 = 0.

The variable “other costs” represents variable costs per unit of product, including fermentation additives, salt, energy, and similar expenses (Q26): Q26: -5.00X1 - 50.00X2 - 60.00X3 - 6.50X4 - 55.00X5 - 5.50X6 - 26.00X7 - 3.00X8 - 6.00X9 - 12.00X10 - 10.00X11 + X26 = 0.

Labor expenditure per unit of product refers to the labor of direct workers in production (Q27): Q27: -0.11X1 - 3.13X2 - 3.90X3 - 0.15X4 - 4.20X5 - 0.64X6 - 2.81X7 - 0.22X8 - 0.49X9 - 1.40X10 - 3.00X11 + X27 = 0.

The balance of finished products connects production lines with activities of finished products (Q28-Q38): Q28: X1 – X28 = 0; Q29: X2 – X29 = 0; Q30: X3 – X30 = 0; Q31: X4 – X31 = 0; Q32: X5 – X32 = 0; Q33: 5.00X6 – X33 = 0; Q34: 1.42857X7 – X34 = 0; Q35: X8 – X35 = 0; Q36: X9 – X36 = 0; Q37: X10 – X37 = 0; Q38: X11 – X38 = 0.

**Criterion function in the milk processing optimization model:** When solving managerial problems in the agri-food complex using linear programming, whether it is in primary agricultural production or the food industry, the most common criterion for maximization is net income or gross margin, in line with the works of authors such as Andrić (1969), Mirić and Krstić (1969), Munčan and colleagues (1991, 1998, 2003), Bulatović (1996), Bogavac Violeta (1996), Sredojević Zorica (2002), Mirjanić and colleagues (2008), Vico (2012), and Jandrić Mersida (2019).

Net income is defined as the difference between total revenue and variable costs and is used as the criterion in this research. In this approach, the coefficients in the criterion function for finished products are positive, representing the selling price per unit of product, while the coefficients for inputs are negative, reflecting the purchase price per unit of input. For other activities in the criterion function, the coefficients have a value of zero.

**Model solution:** Solving a model that includes inputs and outputs in the criterion function has several advantages, such as faster and simpler interpretation of results, enabling separate observation of related products, especially for post-optimal analysis, and facilitating experimentation within the model. By reading the values of activities representing finished products directly from the model, the production structure is obtained.

The optimal production structure of milk in the dairy (Jandrić, 2019) includes eight products, five main and three additional ones. Clotted cream as a main product and two
by-products obtained in the process of clotted cream production are not included in the optimal solution. The largest share of revenue comes from filo cheese pie (26.72%), followed by yogurt 2.8% fat (22.11%) and white cheese slices 45% fat (18.15%). The optimal assortment also includes pasteurized milk 3.2% fat, sour cream 12% fat, pepper in cream 35% fat, fermented whey 0.5% fat, and urda cheese. The total revenue is 352,702.93 RSD, while the net income is 145,859.42 RSD. According to Jandrić’s analysis (2019), the calculation of input values in the dairy shows the percentage share of each item in total variable costs (VC). The largest portion of costs is attributed to raw milk, accounting for 67.8% of total costs. Other significant costs include “Other costs” (14.4%) and various types of packaging for products. Labor costs account for 3.9% of total costs.

**Analysis of the obtained solutions for milk processing optimization:** From the optimal solution, managers in the agrofood complex obtain quantitative indicators of inputs and outputs, as well as valuable information from post-optimal analysis. Post-optimal analysis provides insight into the quality of the optimal solution and assists in making future business decisions.

In the optimal solution, pasteurized milk is projected at a quantity of 400 liters per day, which is the minimum set by a specific constraint. Increasing the selling price by more than 3.60 RSD per liter (7.20%) would change the dairy production structure. The selling price of white cheese slices can be increased infinitely without altering the production structure since it is already at the maximum allowed quantity (128 kg/day). Decreasing the price of white cheese slices by more than 82.99 RSD per kilogram (16.60%) would result in a reduction of daily production quantities. The most significant changes in selling prices required to alter the optimal solution are for urda (increase of 119.10%) and pepper in cream (increase of 40.40%). The largest allowable decreases in prices without impacting the optimal solution are for fermented whey (70.82%), pepper in cream (37.70%), and sour cream (36.22%).

Sensitivity analysis of dairy products reveals that products with maximum allowable production, such as white cheese slices, yogurt 2.8% fat, and sour cream, can infinitely increase prices without affecting the optimal solution. Fermented whey, a by-product of white cheese slices, follows a similar pattern. Paprika in cream, a by-product of 2.8% fat yogurt, can only increase prices by up to 40.40% without changing the optimal solution, due to its connection to pasteurized milk production. Products not included in the optimal solution, such as cream cheese and 0.5% fat yogurt, as well as products with minimum required production, will not undergo production changes even if prices decrease.
Table 1- Sensitivity analysis of coefficients for the group of ‘Finished Products’ activities.

<table>
<thead>
<tr>
<th>Label</th>
<th>Activity Name</th>
<th>Quantity of Activity in Optimal Solution</th>
<th>Value of Coefficient in Criterion Function (RSD)</th>
<th>Permissible Increase (RSD)</th>
<th>Permissible Decrease (RSD)</th>
<th>Permissible Increase (%)</th>
<th>Permissible Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_{28}</td>
<td>Pasteurized milk 3.2% fat</td>
<td>400,00</td>
<td>50,00</td>
<td>3,60</td>
<td>∞</td>
<td>7,20</td>
<td></td>
</tr>
<tr>
<td>X_{29}</td>
<td>White cheese slices 45% fat</td>
<td>128,00</td>
<td>500,00</td>
<td>∞</td>
<td>82,99</td>
<td>16,60</td>
<td></td>
</tr>
<tr>
<td>X_{30}</td>
<td>Clotted cream 45-50% fat</td>
<td>0,00</td>
<td>600,00</td>
<td>6,45</td>
<td>∞</td>
<td>1,08</td>
<td></td>
</tr>
<tr>
<td>X_{31}</td>
<td>Yogurt 2.8% fat</td>
<td>1 200,00</td>
<td>65,00</td>
<td>∞</td>
<td>5,00</td>
<td>7,70</td>
<td></td>
</tr>
<tr>
<td>X_{32}</td>
<td>Filo cheese pie 20% fat</td>
<td>171,33</td>
<td>550,00</td>
<td>47,64</td>
<td>3,23</td>
<td>8,66</td>
<td>0,59</td>
</tr>
<tr>
<td>X_{33}</td>
<td>Sour cream 12% fat</td>
<td>1 000,00</td>
<td>55,00</td>
<td>∞</td>
<td>19,92</td>
<td>36,22</td>
<td></td>
</tr>
<tr>
<td>X_{34}</td>
<td>Pepper in cream 35% fat</td>
<td>75,46</td>
<td>260,00</td>
<td>105,03</td>
<td>97,25</td>
<td>40,40</td>
<td>37,40</td>
</tr>
<tr>
<td>X_{35}</td>
<td>Fermented whey 0.5% fat</td>
<td>500,00</td>
<td>30,00</td>
<td>∞</td>
<td>21,24</td>
<td>70,82</td>
<td></td>
</tr>
<tr>
<td>X_{36}</td>
<td>Yogurt 0.5% fat</td>
<td>0,00</td>
<td>60,00</td>
<td>0,81</td>
<td>∞</td>
<td>1,34</td>
<td></td>
</tr>
<tr>
<td>X_{37}</td>
<td>Baker’s cheese</td>
<td>0,00</td>
<td>120,00</td>
<td>3,23</td>
<td>∞</td>
<td>2,69</td>
<td></td>
</tr>
<tr>
<td>X_{38}</td>
<td>Urda cheese</td>
<td>68,53</td>
<td>100,00</td>
<td>119,10</td>
<td>8,06</td>
<td>119,10</td>
<td>8,06</td>
</tr>
</tbody>
</table>

Source: Jandrić, 2019

Table 2 - Sensitivity Analysis of Constraints

<table>
<thead>
<tr>
<th>Label</th>
<th>Name of Constraint</th>
<th>Defined Value</th>
<th>Final Value</th>
<th>Shadow Price</th>
<th>Allowable Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>O_{5}</td>
<td>Maximum Daily Capacity White Cheese kg</td>
<td>128,00</td>
<td>128,00</td>
<td>82,99</td>
<td>219,30</td>
</tr>
<tr>
<td>O_{6}</td>
<td>Market Limitation Sour Cream 1000 pcs/day</td>
<td>200,00</td>
<td>200,00</td>
<td>99,61</td>
<td>565,38</td>
</tr>
<tr>
<td>O_{7}</td>
<td>Market Limitation Yogurt 2.8% mm max. 1200 l/day</td>
<td>1 200,00</td>
<td>1 200,00</td>
<td>5,00</td>
<td>1 631,03</td>
</tr>
<tr>
<td>O_{8}</td>
<td>Market Limitation Yogurt 0.5% mm max. 800 l/day</td>
<td>800,00</td>
<td>0,00</td>
<td>0,00</td>
<td></td>
</tr>
<tr>
<td>O_{9}</td>
<td>Market Limitation Pasteurized Milk 3.2% mm min 400 l/day</td>
<td>400,00</td>
<td>400,00</td>
<td>-3,60</td>
<td>1 631,03</td>
</tr>
</tbody>
</table>

Source: Jandrić, 2019
Conclusion

The optimization results have shown which dairy products should be produced in optimal quantities to achieve the best financial results. This will enable more efficient production planning and resource management. The analysis has highlighted the products that contribute the most to total revenue, as well as those most sensitive to price changes. Management can focus on products with the greatest growth and profit potential. Sensitivity analysis has revealed where products are most sensitive to price and resource changes, allowing for better risk management and exploitation of potential market opportunities. Based on the results, management can make informed decisions about changes in the production program, adjusting pricing strategies, and seeking new markets. The analysis opens up opportunities for further research to improve production processes, reduce costs, and increase profitability. These conclusions can assist management in making strategic decisions and achieving long-term success in the dairy business.

Conflict of interests

The authors declare no conflict of interest.

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